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# A FACTOR ANALYSIS OF THE INFORMATION AND ARITHMETIC ITEMS OF THE WAIS[1, 2]

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Several reports have described the application of conventional item-analysis procedures to the Wechsler subtests (6, 7, 9, 12, 13). That these results have not been completely consistent has readily been attributed to differences in the samples studied by different investigators. It may be noted, however, that *if* the items of a particular subtest really do all measure the same thing then no amount of difference in the sampling of people can bring about a significant change in the apparent order of difficulty of the items. Where such changes in the order of difficulty have been observed it is clear that the items must depend reliably on more than one dimension of performance, at least on two dimensions and possibly on as many dimensions as there are items.

In particular, the multidimensional character of the Picture Completion (PC) subtest has recently been explicitly demonstrated (16), with the isolation of three common factors from the 20 items that were studied. At the same time, evidence was adduced showing that these three factors could be interpreted in terms of clinically distinct processes, and that they were differentially related to certain of the subtests other than PC. Given such encouragement, it appears pertinent to investigate the factorial composition of at least some of the remaining Wechsler subtests, in order to provide a better understanding of what is measured by the scores currently being obtained (21) by adding together item responses of heterogeneous content.

The present report is concerned primarily with the Information (I) and Arithmetic (A) subtests of the WAIS.[3] We chose to treat more than one of the subtests at a time, where possible, in order to reduce the number of factor analyses required to cover all the subtests. We chose to treat I and A together,

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[2]This report may be regarded as the fourth in a series (15, 16, 18) concerned with the general question of relationships between aptitude and achievement tests, on the one hand, and various measures of personality on the other. The Wechsler provides a useful starting point for studies in this area since it appears to span a number of representative ability dimensions, and is widely employed clinically in the understanding of personality disorders. This report contributes to the overall series by providing additional information about six of the dimensions measured by the Wechsler.

[3]It has been suggested that "marker tests" should be included if a study such as this is to succeed in finding out what the WAIS really measures. There are three reasons why we have not adopted such a strategy at this stage of our investigation. (a) The necessary data are not currently available to us, and would be relatively difficult to obtain. (b) In terms of *common* factors, tests cannot possibly exhibit a better simple structure than items. (c) From the viewpoint of those whose experience is with the Wechsler, individual items may serve as even more meaningful "markers" than any collection of less familiar "factor-pure" tests.

in particular, because we anticipated the presence of at least one factor common to both subtests, a dimension of "numerical information." Also, neither I nor A is seriously complicated by the problem of responses for which partial credit is allowed. Comprehension (C) and Similarities (S), which share the latter problem, will be treated together in a subsequent report.[4]

## PROCEDURE

The sample of *S*s was the same as that used twice previously (15, 16), comprising a combined group of 228 male college and college-preparatory students. Separate scores for 51 items were obtained from our original WAIS record forms, including six difficult I items and two difficult A items that had been employed with our *S*s in an effort to provide additional ceiling for these two subtests, as well as all the regular I and A items (21). (The non-standard items are described in Table 1.) At the easy end of the scale, five I items and

TABLE 1

SUPPLEMENTARY INFORMATION AND ARITHMETIC ITEMS (WITH THEIR ANSWERS)

| Item No. | Item |
|---|---|
| I -30 | Who wrote *Principia Mathematica?* (Newton . . . Russell . . . Russell and Whitehead) |
| I -31 | How high is Mt. Everest? [28,000 to 30,000 ft. (Any answer in this range is acceptable.)] |
| I -32 | What is the capital of Australia? (Canberra) |
| I -33 | What is an Habeas Corpus? [A legal writ requiring that evidence be produced to justify detention of a prisoner. (Answer must be clearly distinct from definition of "bail.")] (From WB-I) |
| I -34 | What is Archimedes' Principle? [The bouyant force of a fluid on an immersed body is equal to the weight of fluid displaced. (Answer must go beyond the special case of an object floating in water; the weight of the *object* is relevant only for a floating object.)] |
| I -35 | What is an "occluded front?" [A weather situation in which a cold front has *overtaken* a warm front. (A front is *any* boundary between a cold air mass and a warm air mass, and normally may be "cold," "warm," or "stationary" depending on its direction of movement. None of these is an occluded front.)] |
| A-15 | What is the sum of all the odd numbers between zero and twenty? (100) |
| A-16 | If a train goes 150 yards in 10 seconds, how many feet can it go in one fifth of a second? (9 ft.) (Administered visually as in WB-I.) |

three A items were passed by every *S* in our sample, leaving only 43 items available for intercorrelation and factor analysis. In the case of those A items for which extra time-credits may be allowed for fast performance we simply

[4]Saunders, D. R. A factor analysis of Comprehension and Similarities items from the WAIS, in preparation.

| Item | I-6 | I-11 | I-12 | I-13 | I-14 | I-16 | I-17 | I-18 | I-19 | I-20 |
|---|---|---|---|---|---|---|---|---|---|---|
| I - 6 | — | -16 | 13 | 04 | 26 | 21 | 26 | 23 | 02 | 16 |
| I -11 | -16 | — | 30 | 02 | -08 | -20 | 14 | 13 | -16 | -17 |
| I -12 | 13 | 30 | — | 19 | -00 | 26 | 39 | 42 | 14 | 20 |
| I -13 | 04 | 02 | 19 | — | -09 | 11 | 08 | 09 | 46 | 14 |
| I -14 | 26 | -08 | -00 | -09 | — | 28 | 18 | 16 | -02 | 17 |
| I -16 | 21 | -20 | 26 | 11 | 28 | — | 38 | 42 | 24 | 40 |
| I -17 | 26 | 14 | 39 | 08 | 18 | 38 | — | 41 | 03 | 41 |
| I -18 | 23 | 13 | 42 | 09 | 16 | 42 | 41 | — | 31 | 51 |
| I -19 | 02 | -16 | 14 | 46 | -02 | 24 | 03 | 31 | — | 20 |
| I -20 | 16 | -17 | 20 | 14 | 17 | 40 | 41 | 51 | 20 | — |
| I -21 | 23 | 15 | 26 | -03 | 56 | 31 | 20 | 24 | 15 | 51 |
| I -22 | 25 | -19 | 06 | 10 | 25 | 29 | 32 | 13 | 05 | -01 |
| I -23 | 35 | -06 | 23 | 37 | 18 | 43 | 18 | 29 | 44 | 36 |
| I -24 | 30 | -04 | 19 | 05 | 29 | 40 | 31 | 35 | 25 | 40 |
| I -25 | 31 | 08 | 24 | 18 | 03 | -01 | 12 | 31 | 34 | 09 |
| I -26 | 21 | 02 | 45 | 32 | 17 | 66 | 33 | 51 | 13 | 38 |
| I -27 | 22 | 06 | 11 | 15 | 12 | 51 | 09 | 02 | 01 | -12 |
| I -29 | -09 | 06 | 13 | 20 | 20 | 38 | 23 | 12 | 16 | 08 |
| I -31 | 40 | -07 | 31 | 35 | 19 | 47 | 19 | 23 | 23 | 39 |
| A- 5 | -08 | -09 | 06 | 09 | 29 | 03 | -01 | 15 | -01 | 11 |
| A- 6 | -09 | -00 | 30 | 16 | 32 | 31 | 24 | 35 | 02 | 28 |
| A- 8 | 07 | 03 | 08 | 11 | 10 | -16 | -05 | 21 | 12 | 11 |
| A- 9 | -05 | -05 | 14 | 46 | 24 | 17 | -05 | 16 | 29 | 40 |
| A-10 | 08 | 22 | -09 | -05 | 30 | 02 | 05 | 15 | 04 | 05 |
| A-11 | 11 | 06 | 13 | 08 | 06 | 01 | 10 | 19 | 26 | 19 |
| A-12 | 01 | -14 | 02 | 42 | 23 | 20 | 07 | -07 | 22 | 31 |
| A-13 | 08 | -20 | 14 | 29 | 09 | 22 | 14 | 11 | 13 | 24 |
| A-14 | -02 | 15 | 17 | 13 | 29 | 26 | 03 | 21 | 16 | 30 |
| A-15 | 00 | 14 | -02 | 36 | 27 | 07 | 05 | 13 | 01 | 03 |
| A-16 | 39 | -08 | 14 | 35 | 37 | 22 | 23 | 31 | 30 | 35 |

TABLE 2

TETRACHORIC INTERCORRELATION MATRIX ($N = 228$)

| I-21 | I-22 | I-23 | I-24 | I-25 | I-26 | I-27 | I-29 | I-31 | A-5 | A-6 | A-8 | A-9 | A-10 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 25 | 35 | 30 | 31 | 21 | 22 | –09 | 40 | –08 | –09 | 07 | –05 | 08 |
| 15 | –19 | –06 | –04 | 08 | 02 | 06 | 06 | –07 | –09 | –00 | 03 | –05 | 22 |
| 26 | 06 | 23 | 19 | 24 | 45 | 11 | 13 | 31 | 06 | 30 | 08 | 14 | –09 |
| –03 | 10 | 37 | 05 | 18 | 32 | 15 | 20 | 35 | 09 | 16 | 11 | 46 | –05 |
| 56 | 25 | 18 | 29 | 03 | 17 | 12 | 20 | 19 | 29 | 32 | 10 | 24 | 30 |
| 31 | 29 | 43 | 40 | –01 | 66 | 51 | 38 | 47 | 03 | 31 | –16 | 17 | 02 |
| 20 | 32 | 18 | 31 | 12 | 33 | 09 | 23 | 19 | –01 | 24 | –05 | –05 | 05 |
| 24 | 13 | 29 | 35 | 31 | 51 | 02 | 12 | 23 | 15 | 35 | 21 | 16 | 15 |
| 15 | 05 | 44 | 25 | 34 | 13 | 01 | 16 | 23 | –01 | 02 | 12 | 29 | 04 |
| 51 | –01 | 36 | 40 | 09 | 38 | –12 | 08 | 39 | 11 | 28 | 11 | 40 | 05 |
| — | 16 | 41 | 15 | 27 | 28 | 05 | –09 | 20 | 36 | 33 | 05 | 35 | 21 |
| 16 | — | 09 | 26 | 16 | 12 | 41 | 39 | 16 | 08 | 06 | 18 | –05 | 09 |
| 41 | 09 | — | 35 | 35 | 50 | 03 | 08 | 44 | 09 | 27 | 36 | 35 | 19 |
| 15 | 26 | 35 | — | 27 | 59 | 23 | 26 | 26 | 04 | 12 | 29 | –02 | 13 |
| 27 | 16 | 35 | 27 | — | 42 | 00 | –06 | 40 | 01 | 11 | 13 | 33 | 23 |
| 28 | 12 | 50 | 59 | 42 | — | 25 | 25 | 57 | 23 | 26 | 14 | 27 | 27 |
| 05 | 41 | 03 | 23 | 00 | 25 | — | 52 | 21 | –24 | 15 | –17 | 02 | 01 |
| –09 | 39 | 08 | 26 | –06 | 25 | 52 | — | 05 | –16 | 02 | –31 | –14 | –05 |
| 20 | 16 | 44 | 26 | 40 | 57 | 21 | 05 | — | 17 | 45 | 24 | 21 | 22 |
| 36 | 08 | 09 | 04 | 01 | 23 | –24 | –16 | 17 | — | 20 | 37 | 24 | 57 |
| 33 | 06 | 27 | 12 | 11 | 26 | 15 | 02 | 45 | 20 | — | –19 | 22 | 28 |
| 05 | 18 | 36 | 29 | 13 | 14 | –17 | –31 | 24 | 37 | –19 | — | 07 | 07 |
| 35 | –05 | 35 | –02 | 33 | 27 | 02 | –14 | 21 | 24 | 22 | 07 | — | 13 |
| 21 | 09 | 19 | 13 | 23 | 27 | 01 | –05 | 22 | 57 | 28 | 07 | 13 | — |
| 05 | 11 | 09 | 19 | 11 | 17 | –02 | –03 | 12 | 39 | 15 | 33 | 22 | 30 |
| 37 | 23 | 31 | 25 | 20 | 32 | 07 | 10 | 31 | 38 | –01 | 26 | 34 | 10 |
| 29 | 11 | 16 | 31 | 11 | 26 | 03 | 12 | 14 | 24 | 26 | 32 | 34 | 26 |
| 20 | 05 | 20 | 11 | 20 | 32 | 18 | 24 | 40 | 32 | 09 | 39 | 32 | 24 |
| 27 | –01 | 09 | 01 | 14 | 12 | 07 | 04 | 16 | 17 | –17 | 33 | 29 | 31 |
| 36 | 16 | 32 | 36 | 23 | 23 | –18 | –14 | 41 | 44 | 18 | 32 | 28 | 22 |

| A-11 | A-12 | A-13 | A-14 | A-15 | A-16 | $P_1$ |
|---|---|---|---|---|---|---|
| 11 | 01 | 08 | –02 | 00 | 39 | .93 |
| 06 | –14 | –20 | 15 | 14 | –08 | .80 |
| 13 | 02 | 14 | 17 | –02 | 14 | .82 |
| 08 | 42 | 29 | 13 | 36 | 35 | .83 |
| 06 | 23 | 09 | 29 | 27 | 37 | .63 |
| 01 | 20 | 22 | 26 | 07 | 22 | .86 |
| 10 | 07 | 14 | 03 | 05 | 23 | .58 |
| 19 | –07 | 11 | 21 | 13 | 31 | .78 |
| 26 | 22 | 13 | 16 | 01 | 30 | .52 |
| 19 | 31 | 24 | 30 | 03 | 35 | .56 |
| 05 | 37 | 29 | 20 | 27 | 36 | .72 |
| 11 | 23 | 11 | 05 | –01 | 16 | .83 |
| 09 | 31 | 16 | 20 | 09 | 32 | .64 |
| 19 | 25 | 31 | 11 | 01 | 36 | .81 |
| 11 | 20 | 11 | 20 | 14 | 23 | .48 |
| 17 | 32 | 26 | 32 | 12 | 23 | .54 |
| –02 | 07 | 03 | 18 | 07 | –18 | .40 |
| –03 | 10 | 12 | 24 | 04 | –14 | .31 |
| 12 | 31 | 14 | 40 | 16 | 41 | .25 |
| 39 | 38 | 24 | 32 | 17 | 44 | .92 |
| 15 | –01 | 26 | 09 | –17 | 18 | .91 |
| 33 | 26 | 32 | 39 | 33 | 32 | .90 |
| 22 | 34 | 34 | 32 | 29 | 28 | .86 |
| 30 | 10 | 26 | 24 | 31 | 22 | .74 |
| — | 04 | 10 | 22 | 26 | 27 | .70 |
| 04 | — | 26 | 41 | 28 | 43 | .69 |
| 10 | 26 | — | 32 | 36 | 41 | .37 |
| 22 | 41 | 32 | — | 46 | 24 | .36 |
| 26 | 28 | 36 | 46 | — | 44 | .38 |
| 27 | 43 | 41 | 24 | 44 | — | .37 |

noted whether or not the correct answer was reached within the *longest* time limit, thus minimizing the importance of speed for these items.

A 43 × 43 tetrachoric intercorrelation matrix was computed using an IBM 650 procedure developed by Tucker (1). This procedure "flagged" a number of its own results as relatively unreliable, on grounds of too few cases in one or another of the cells of the two-by-two table from which the tetrachoric correlation was being estimated. Thirteen additional items had to be set aside because they were involved in more than one such unreliable correlation estimate. All 13 of these items had been passed by less than 5% or by more than

TABLE 3

PRINCIPAL COMMON FACTOR MATRIX

| Item | $F_1$ | $F_2$ | $F_3$ | $F_4$ | $F_5$ | $F_6$ | $h^2$ |
|---|---|---|---|---|---|---|---|
| I - 6 | 350 | 158 | –176 | 013 | –466 | 096 | 405 |
| I -11 | –030 | –016 | –119 | –039 | 464 | 398 | 390 |
| I -12 | 401 | 226 | –210 | 197 | 278 | 194 | 410 |
| I -13 | 428 | –074 | 442 | 438 | 134 | –054 | 597 |
| I -14 | 438 | –040 | –051 | –540 | –095 | –189 | 532 |
| I -16 | 606 | 531 | 064 | –103 | –002 | –215 | 710 |
| I -17 | 402 | 331 | –245 | –052 | –015 | 118 | 348 |
| I -18 | 554 | 177 | –376 | 108 | 108 | 202 | 543 |
| I -19 | 391 | –008 | 154 | 416 | –043 | –047 | 354 |
| I -20 | 581 | 034 | –257 | 107 | –012 | –247 | 478 |
| I -21 | 553 | –122 | –212 | –233 | 074 | –274 | 501 |
| I -22 | 302 | 266 | 199 | –252 | –318 | 119 | 381 |
| I -23 | 626 | 024 | –040 | 274 | –096 | –094 | 486 |
| I -24 | 544 | 238 | –072 | –019 | –285 | 198 | 479 |
| I -25 | 427 | –044 | –076 | 238 | –014 | 155 | 271 |
| I -26 | 726 | 256 | –058 | 093 | 113 | 126 | 634 |
| I -27 | 211 | 561 | 419 | –234 | 079 | 094 | 605 |
| I -29 | 199 | 565 | 413 | –180 | 130 | 110 | 591 |
| I -31 | 640 | 076 | –012 | 110 | –025 | –014 | 429 |
| A- 5 | 382 | –526 | –084 | –328 | 063 | –006 | 542 |
| A- 6 | 407 | 140 | –276 | –119 | 316 | –268 | 447 |
| A- 8 | 328 | –466 | 041 | 086 | –256 | 398 | 558 |
| A- 9 | 477 | –297 | 144 | 194 | 270 | –330 | 556 |
| A-10 | 345 | –284 | –107 | –354 | 153 | 143 | 381 |
| A-11 | 320 | –236 | –058 | –034 | 038 | 251 | 227 |
| A-12 | 501 | –238 | 379 | –019 | –125 | –184 | 501 |
| A-13 | 459 | –184 | 147 | –047 | –009 | –038 | 270 |
| A-14 | 504 | –199 | 275 | –167 | 227 | 155 | 473 |
| A-15 | 362 | –377 | 323 | –146 | 118 | 190 | 449 |
| A-16 | 628 | –348 | –047 | 011 | –262 | –021 | 587 |
| $\lambda$ | 6.386 | 2.489 | 1.497 | 1.437 | 1.202 | 1.126 | 14.135 |

95% of our sample. The correlation matrix for the remaining 30 items is reported in Table 2, including the proportions of our sample passing them.

The factor analysis proper was carried out by an iterative procedure that we have described elsewhere (19); it resulted in the determination of a six-factor solution. The unrotated factor matrix and communalities are reported in Table 3. The latent roots for the discarded factors ranged from +0.784 to —0.757, exhibiting a symmetrical distribution around zero.

Rotation was carried out initially to the raw quartimax position, and then to a series of positions defined by successively improved hypothesis matrices (17). The final orthogonal rotation is reported in Table 4. Parentheses are

TABLE 4

ORTHOGONALLY ROTATED FACTOR MATRIX

| Item | I | II | III | IV | V | VI | $h^2$ |
|---|---|---|---|---|---|---|---|
| I - 6 | (625) | 080 | 059 | –057 | 017 | –019 | 405 |
| I -11 | [–321] | (406) | 019 | –086 | –176 | (289) | 390 |
| I -12 | 095 | (602) | 110 | 160 | –007 | 015 | 410 |
| I -13 | 048 | 055 | 144 | (754) | –009 | 046 | 597 |
| I -14 | (241) | –018 | 187 | –151 | (599) | (240) | 532 |
| I -16 | (330) | (292) | (548) | 154 | (388) | –201 | 710 |
| I -17 | (320) | (424) | 216 | –079 | 111 | –024 | 348 |
| I -18 | (334) | (639) | 033 | 066 | 107 | 079 | 543 |
| I -19 | (236) | 126 | 014 | (527) | –018 | –062 | 354 |
| I -20 | (348) | (342) | –065 | 229 | (416) | –098 | 478 |
| I -21 | 205 | 224 | –038 | 077 | (621) | (126) | 501 |
| I -22 | (397) | –089 | (440) | –058 | 065 | 119 | 381 |
| I -23 | (427) | (272) | 016 | (432) | 207 | –024 | 486 |
| I -24 | (579) | (248) | (254) | 058 | 054 | 107 | 479 |
| I -25 | (285) | (299) | –043 | (288) | –010 | 127 | 271 |
| I -26 | (354) | (534) | (299) | (295) | 191 | 105 | 634 |
| I -27 | 017 | 043 | (776) | 031 | 015 | –011 | 605 |
| I -29 | –023 | 085 | (761) | 062 | –020 | –025 | 591 |
| I -31 | (374) | (305) | 144 | (334) | 239 | 082 | 429 |
| A- 5 | 078 | 014 | –223 | 049 | (448) | (531) | 542 |
| A- 6 | –013 | (431) | 062 | 031 | (498) | –094 | 447 |
| A- 8 | (374) | –022 | –235 | 219 | –136 | (544) | 558 |
| A- 9 | –056 | 115 | –113 | (585) | (422) | 078 | 556 |
| A-10 | 034 | 170 | –036 | –062 | (321) | (492) | 381 |
| A-11 | 149 | 185 | –083 | 109 | 045 | (386) | 227 |
| A-12 | (234) | –199 | 123 | (475) | (334) | (232) | 501 |
| A-13 | 188 | 029 | 061 | (300) | (270) | (259) | 270 |
| A-14 | 007 | 129 | 217 | (320) | (239) | (500) | 473 |
| A-15 | –003 | –054 | 076 | (311) | (143) | (569) | 449 |
| A-16 | (508) | 045 | –165 | (304) | (321) | (322) | 587 |
| $\Sigma a^2$ | 2.648 | 2.338 | 2.206 | 2.494 | 2.383 | 2.063 | 14.135 |

used in Table 4 to indicate which loadings were specified as non-zero in the final hypothesis matrix, which was used to compute the table.

It is evident from a set of factor plots based on Table 4 that certain of the factor pairs are oblique. This probably accounts for the necessity of employing a relatively complex pattern to obtain a good orthogonal approximation of the true oblique factor structure, using the rotation procedure followed. However, it was found possible to take account of the most compelling obliquenesses by executing just four single plane rotations, affecting the loadings on Factors III, V, and VI. These rotations may be summarized in the following transformation matrix:

| | II | III | V | VI |
|---|---|---|---|---|
| II | 1.000 | –140 | 0 | 0 |
| III | 0 | 949 | 0 | 0 |
| V | 0 | 081 | 958 | 287 |
| VI | 0 | 272 | 287 | 958 |

TABLE 5

FACTOR LOADINGS FROM DWYER EXTENSION

| Item | I | II | III | IV | V | VI | $h^2$ |
|---|---|---|---|---|---|---|---|
| I - 5 | 070 | –358 | 206 | 344 | 060 | 460 | 509 |
| I - 7 | 102 | 057 | 551 | 131 | 508 | 503 | 845 |
| I - 8 | –120 | 350 | 546 | 137 | 319 | 119 | 570 |
| I - 9 | –154 | –354 | –055 | 310 | –632 | –402 | 809 |
| I -10 | 314 | –208 | –731 | –584 | –074 | –191 | 1.059 |
| I -15 | 120 | 029 | 320 | 157 | 206 | 312 | 282 |
| I -28 | 074 | 124 | 125 | 279 | 219 | 117 | 176 |
| I -30 | 153 | 039 | 742 | 259 | 597 | 461 | 1.212 |
| I -32 | 078 | 205 | 515 | –004 | 482 | 382 | 692 |
| I -33 | –110 | 468 | 723 | 353 | 603 | 617 | 1.623 |
| I -34 | 094 | 059 | 552 | 251 | 342 | 468 | 716 |
| I -35 | 349 | –343 | 605 | –026 | 547 | 798 | 1.542 |
| A- 7 | –068 | 381 | 394 | 534 | 282 | 428 | 853 |
| I - 5 | 069 | –355 | 204 | 338 | 059 | 456 | |
| I - 7 | 078 | 044 | 424 | 101 | 391 | 387 | |
| I - 8 | –112 | 328 | 511 | 128 | 299 | 111 | |
| I - 9 | –121 | –278 | –043 | 244 | –497 | –316 | |
| I -10 | 216 | –143 | –502 | –401 | –051 | –131 | |
| I -15 | 160 | 039 | 426 | 209 | 274 | 415 | |
| I -28 | 125 | 209 | 211 | 470 | 369 | 197 | |
| I -30 | 098 | 025 | 477 | 166 | 383 | 296 | |
| I -32 | 066 | 174 | 438 | –003 | 410 | 325 | |
| I -33 | –061 | 260 | 401 | 196 | 335 | 342 | |
| I -34 | 079 | 049 | 461 | 210 | 286 | 391 | |
| I -35 | 199 | –195 | 345 | –015 | 311 | 454 | |
| A- 7 | –052 | 292 | 302 | 409 | 216 | 328 | |

Following the completion of orthogonal rotation, estimates were made of the rotated factor loadings of the 13 items that had been set aside at the beginning of the factor analysis, by using a modification of Dwyer's extension method (4). For any one of the 13 additional items, these estimates depend only on the correlations of the item with the 30 items that stayed in the factor analysis, thereby making minimal use of the tetrachoric correlations judged to be unreliable, and containing the effect of any seriously erroneous correlation estimate within a single item. The estimated factor loadings and communalities for these items are shown in Table 5, in the top part of the table.

In view of the extra variance contributed by the poorly estimated correlations, it is not surprising that several of the estimated communalities in Table 5 are greater than unity. The average communality in Table 4, however, is only .47. In order to place the 13 items added by extension on a more comparable basis for the discussion to follow, we have arbitrarily but proportionally adjusted the loadings of these items so they yield a communality of exactly 0.5. These adjusted factor loadings are shown in the bottom portion of Table 5.

## Discussion

The mere fact that six distinct factors have emerged from this factor analysis of items taken from the I and A subtests of the WAIS represents ample reward for having undertaken the analysis. Consideration of Table 4 will show that all six of these factors are involved in Information, and that at least five of them may be involved in Arithmetic. However, the first four factors can be regarded as being primarily related to I, and the last one can be regarded as primarily A. The "numerical information" factor whose existence was anticipated appears as Factor V.

*Factor I.*—The following items from Table 4 have loadings of 0.35 or more (none of the items in Table 5 has a loading on this factor):

| | | |
|---|---|---|
| .625 | I - 6 | Name four men who have been presidents of the United States since 1900. |
| .579 | I -24 | Who wrote the *Iliad?* |
| .508 | A-16 | If a train goes 150 yards in 10 seconds, how many feet can it go in one fifth of a second? |
| .427 | I -23 | At what temperature does water boil? |
| .397 | I -22 | What is the main theme of the Book of Genesis? |
| .374 | I -31 | How high is Mt. Everest? |
| .374 | A- 8 | How many hours will it take a man to walk 24 miles at the rate of three miles an hour? |
| .354 | I -26 | What is the Koran? |
| .348 | I -20 | What is the population of the United States? |

These items call for relatively *non*-specific bits of what we shall term *General Information.* While this factor is about as broad as any of the six in terms of the number of loadings that exceed 0.35, it is relatively lacking in items with high loadings that approach factorial purity. The only item that does not appear to be meaningfully related to at least one other factor is Item I-6, and this item has two quite distinct properties both of which are held in common with most of the other items on this factor.

As of today, the United States has had 10 different presidents since 1900, any four of whom may be named for full credit on Item I-6. Thus, there are 210 different correct answers to this particular item. In this sense, the item may be said to be quite non-specific. In a similar vein, Items I-17,[5] I-20, and I-31 all call for judgments which will be counted as correct anywhere within a fairly broad range of possible answers. Items I-16,[5] I-22, and I-26 may be answered in a variety of ways without losing credit. Item I-23 definitely has two different answers, depending on whether the Centigrade or the Fahrenheit scale is referred to. So does Item A-16, since, if *S* initially states the answer is "3 yards," he still may receive full credit for stating that it is "9 feet." Only Items A-8 and I-24 fail to bear an obvious relationship to this factor in terms of non-specificity of required response, and rationalizations could be offered even for these items. In effect, for Factor I, the item form for most of these items might as well have been "Tell me something about . . . ," giving full credit for any relevant and correct reply.

The other feature of Item I-6 is one that is shared particularly clearly with Item I-24, namely, that the answer has been true for a long time. Indeed, in view of the relatively broad limits used in scoring Item I-20, the only item on this factor that may not be appropriately approached in historical perspective is I-11.[6] The latter item appears with a negative loading, although the item is positively correlated with Factors II and VI. It would seem that the average height of women may have been increasing sufficiently rapidly, in relation to the range considered correct in scoring, to cause underestimation on the part of some who look too far back for a basis for answering this item.

Of course, we could note that the answers related to Factor III have been true for a long time, and we would judge that the same could be said for each of the factors except Factor II. Thus, it does not add very much to our understanding of Factor I to make this statement about it. On the other hand, Factor

[5]The choice of a limit such as 0.35 for acceptance of a factor loading is quite arbitrary. It is based on the belief that loadings above this value *in this study* are consistently meaningful. A number of significant loadings must fall short of such a limit, however, and we shall feel free to cite items with loadings of less than 0.35 on a factor when it seems appropriate to do so. *All* of the loadings in Table 5 will be treated as though they were below the limit set for automatic acceptance.

[6]The loading of this item is just below the arbitrary limit of 0.35, but would be *in*creased in magnitude if Factor I were rotated oblique to Factor II.

I appears to be unique in its property of non-specificity, discussed above, and we suspect that it is more fruitful to emphasize this aspect of the factor. This is what we mean to do by referring to the factor as "general information."

If we had given the Vocabulary (V) subtest to *Ss* used in this study,[7] it would certainly have been appropriate to include at least some V items in this analysis. We do know, however, that I and V are consistently and substantially intercorrelated (21). Except for words whose special meanings make them relevant to Factor II or III or IV, e.g., Item I-28, we would expect Factor I to be primarily responsible for this relationship. Indeed, the scoring criteria for the Vocabulary subtest (21) open the door for non-specificity in response just as widely as do the Information items cited above. The same is also true of many Comprehension and Similarities items and can account for the relation of these tests to the Information Factor in our original analysis of these data [Factor II in (15)]. Thus, the feature of non-specificity appears to be a potentially important consideration in almost any open-ended verbal item that must be objectively evaluated. It appears that Factor I may be a general factor across such items, and that it is actually more dependent on item form than on item content!

We also know that Vocabulary, Information, and Comprehension are among the subtests most resistant to decline in old age (22), and again it seems possible to provide an explanation primarily in terms of this factor. In this connection Riegel's comparison of the effects of aging on five verbal item types (11) is of interest. Using a multiple-choice item format throughout, he found least decline for Synonyms, and successively greater decline for Antonyms, Selections, Classifications, and Analogies. Riegel accounted for these findings by noting that the class of possible synonyms for a word was much smaller than the class of possible analogies for a pair of words, so that the habit associating a word with its correct synonym is relatively strong if not often actually over-learned, and is therefore more resistant to decline. Consistently with this hypothesis, we may readily imagine that if Riegel's items were put into open-ended form and we asked an *S*, for example, to "Tell me something about 'flesh,'" it would be much more reasonable and likely for him to reply, "It means the same as 'meat,'" than for him to reply, "It is to a 'man' as 'wood' is to a 'tree.'" In other words, when we use open-ended, non-specific items to measure the amount of General Information that an *S* can recall, we are probably depending primarily on associations that are relatively over-learned, and for this reason relatively non-subject to decline.

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[7]As has previously been reported (15), Vocabulary was omitted primarily in order to save time in test administration, since its high correlation with Information made it seem relatively unlikely that this would lose a factor.

At the same time, we are determining *how much* such material *S* has been able to "find room for." If we may assume something approaching saturation as regards opportunity to learn these items, it is appropriate to regard the resulting score as a good measure of *mental capacity.* Indeed, Merrill and Heathers (8) have proposed using Vocabulary level as a baseline for the interpretation of differential performance on the Wechsler subtests, while Gittinger (14) has proposed for the same purpose a composite based primarily on Information and Comprehension.

*Factor II.*—The following items from Table 4 have loadings above 0.35.

| | | |
|---|---|---|
| .639 | I -18 | Where is Egypt? |
| .602 | I -12 | What is the capital of Italy? |
| .534 | I -26 | What is the Koran? |
| .431 | A- 6 | How many inches are there in two and one-half feet? |
| .424 | I -17 | How far is it from Paris to New York? |
| .406 | I -11 | How tall is the average American woman? |

The items correlating with this factor appear to call for fairly specific bits of information that are useful in understanding *Contemporary Affairs.* Since the authors of the WAIS naturally tried to avoid employing items that would have only transitory interest, this interpretation might seem at first to be an over-generalization. However, we cannot interpret the factor more narrowly, in terms of geographical information for example, without running into difficulty accounting for the utter absence of positive loadings for Items I-9 and I-10 (Table 5). On the other hand, map questions based primarily on places where things have actually been happening do form an appropriate part of most contemporary affairs quizzes. Furthermore, the presence of both Items I-11 and I-20 with consistently signed loadings suggests that the individual who earns a high score on this factor is effectively oriented in the present tense. We would expect it to be appropriate to call him "up-to-date" and "well-informed." Items I-16 and I-21, which also ought to load this factor if our interpretation is correct, do in fact have loadings in excess of 0.2. Obviously, many of the items that would provide the best loadings for this factor, as we interpret it, could not be included in the WAIS, and the factor may be of considerably more practical importance than its variance in this factor analysis would suggest.

*Factor III.*—The following items have loadings above 0.35 according to the *oblique* transformation of Table 4:

| | | |
|---|---|---|
| .729 | I -27 | Who wrote *Faust?* |
| .702 | I -29 | What is the Apocrypha? |
| .468 | I -22 | What is the main theme of the Book of Genesis? |
| .456 | I -16 | What is the Vatican? |

All four of these items call for specific bits of what we may generally term *Cultural Information.* Items dealing with religious and literary themes tend to be most nearly factor-pure for this factor, and this identification will readily account for the loadings shown by Items I-7, I-15, I-24, and I-26. However, a narrow interpretation of the factor in these terms does not suffice to explain the consistent appearance of loadings for other items dealing with scientific and legal matters, such as I-30, I-33, and I-34.

None of the items loading Factor III calls for the sort of information that one will be casually exposed to except in a highly intellectual environment, and the answers will normally be acquired through reading books. It is, therefore, probably appropriate to regard this factor as the residue of a liberal arts education and to regard the individual who earns a high score on the factor as a relatively "educated" man. We might even call him an "intellectual" or an "egg-head" if we wished to emphasize the *relative* lack of practical day-to-day relevance contained in this kind of information. Obviously, the pursuit and acquisition of "cultural information" can be rewarded only in terms of a value-system that de-emphasizes the practical, and only a small fraction of the population can afford this luxury. Only a small fraction of even our present college-oriented sample has put a large effort in this direction, and for this reason most of the highly-loaded items for this factor appear to be relatively quite difficult.

*Factor IV.*—The following items from Table 4 have loadings in excess of 0.35:

| | | |
|---|---|---|
| .754 | I -13 | Why are dark clothes warmer than light-colored clothes? |
| .585 | A- 9 | If a man buys seven two-cent stamps and gives the clerk a half dollar, how much change should he get back? |
| .527 | I -19 | How does yeast cause dough to rise? |
| .475 | A-12 | A man bought some secondhand furniture for two thirds of what it costs new. He paid $400 for it. How much did it cost new? |
| .432 | I -23 | At what temperature does water boil? |

This factor appears to represent primarily the application of *Scientific Information* to the solution of specific problems. While some of these "problems" are trivial and call only for the recitation of the required information, others will require some deductive exercise on *S*'s part. It happens that the most nearly factor-pure items for this factor in the present analysis come from the Information subtest.

Only two Information items (I-5 and I-28) having relatively little variance and only weak raw correlations with this factor were present in WB-I; evidently increased importance was attached to this factor in the development of the

WAIS Information subtest. However, the most consistent utilization of the factor actually appears to be in the solution of relatively difficult Arithmetic items. Every item that is sufficiently complex to require either a division or a sequence of more than one operation for its solution is appreciably correlated with this factor.

*Factor V.*—The following items have loadings above 0.35 according to the oblique transformation of Table 4:

| | | |
|---|---|---|
| .643 | I -14 | When is Washington's birthday? |
| .631 | I -21 | How many senators are there in the United States Senate? |
| .582 | A- 5 | A newsman collected 25 cents from each of six customers. What is the total amount he collected? |
| .450 | A- 6 | How many inches are there in two and one-half feet? |
| .449 | A-10 | A man with $18 spends $7.50. How much does he have left? |
| .427 | A- 9 | If a man buys seven two-cent stamps and gives the clerk a half dollar, how much change should he get back? |
| .400 | A-16 | If a train goes 150 yards in 10 seconds, how many feet can it go in one fifth of a second? |
| .387 | A-12 | A man bought some secondhand furniture for two thirds of what it costs new. He paid $400 for it. How much did it cost new? |
| .372 | A-14 | Eight men can finish a job in six days. How many men will be needed to finish it in a half day? |
| .370 | I -20 | What is the population of the United States? |

The common denominator of this factor may be very clearly expressed as specific *Numerical Information.* Like Factor IV, Factor V is involved in both the Information and Arithmetic subtests. The only Arithmetic item that does not substantially load Factor V is the only Arithmetic item (A-8) that does not require conversion of units for its solution. Knowledge of the number of inches in a foot, cents in a quarter, etc., is one important kind of "numerical information." Looking at the Information items, we may note that those requiring only a rough judgment even if in numerical terms, such as I-17 and I-20, are not as well correlated with this factor as those which require a particular, specific numerical response, such as I-14 and I-21. This factor may be related to memory for detail; in this sense it would complement Factor IV, which we might relate to memory for generalizations.

Sommer's recent discussion (20) of sex differences in the retention of quantitative information seems to involve primarily this factor. Sommer refers particularly to WAIS Items I-17 and I-20 as ones which males do better on than females, and cites the item, "How many teaspoons are there in a table-

spoon?" as one which females do better on. The latter clearly involves nothing more than a conversion of units, and would be expected to load Factor V in the same direction as the other two items according to our interpretation of this factor. Evidently we must be prepared for this factor to be loaded by somewhat different items for men and for women, although in each case we should expect to find loadings for items of information that have a direct and practical utility for the role of the sex. The profile of factor loadings in the present study is "for men only." If role is important in relation to this factor, we should also note that the present loadings are for a "college-oriented" sample.

*Factor VI.*—The following items have loadings above 0.35 according to the oblique transformation of Table 4.

| | | |
|---|---|---|
| .637 | A- 5 | A newsman collected 25 cents from each of six customers. What is the total amount he collected? |
| .586 | A-15 | What is the sum of all the odd numbers between zero and twenty? |
| .563 | A-10 | A man with $18 spends $7.50. How much does he have left? |
| .548 | A-14 | Eight men can finish a job in six days. How many men will be needed to finish it in a half day? |
| .482 | A- 8 | How many hours will it take a man to walk 24 miles at the rate of three miles an hour? |
| .402 | I -14 | When is Washington's birthday? |
| .401 | A-16 | If a train goes 150 yards in 10 seconds, how many feet can it go in one fifth of a second? |
| .383 | A-11 | The price of canned peas is two cans for 31 cents. What is the price of one dozen cans? |

This factor clearly involves skill in performing *Numerical Operations.* It is loaded by every Arithmetic item except A-6; the computational aspect of A-6 may be regarded as minor in comparison with the information required. At the other extreme, the computation in Items A-5 and A-15 far overshadows the information required. The two Information items appearing on Factor VI will depend on the use of computational skill for at least some *S*s; responding to Item I-11 may involve finding an average, while responding to Item I-14 is likely to involve use of a mnemonic device that relates both Lincoln's and Washington's birthdates to a more personally relevant anniversary of some kind.

While there is some tendency for Arithmetic items to appear on every factor, it is clear that Factors V and VI make the most consistent contribution to the total score on Arithmetic throughout the range of item difficulty. Inasmuch as most appearances of the Numerical Operations factor in other studies (2, 3, 5) have included the notion of speed in performing arithmetic problems, it appears safe to assume that the contribution of time-credits (21) to the total score in-

creases the weight of Factor VI relative to Factor V, and makes Factor VI the most important single consideration in understanding the overall score.

Coombs (2) concluded that what has been called "number ability" is basically "characterized by a facility in manipulating a symbolic system according to a specified set of rules." The more practice was provided or the more familiar the symbolism had become, the more performance was shown to depend on the number factor. Performances requiring serial response did not correlate so well with the factor as very simple tasks involving a single operation. However, "number ability" could be detected in tasks that do not involve numbers at all. Rapaport's (10) discussion of the Arithmetic subtest of the WB-I as a measure of "concentration" and Gittinger's (14) interpretation of A as a measure of "ideational self-discipline" are consistent with Coombs' stress on the more fundamental nature of this factor.

It is clear that the present results provide further confirmation for this position. We have already observed that A items requiring serial operations are all loaded on another factor (Factor IV), while Factor VI has its best correlations with *simple* addition or multiplication items like A-5 and A-15. In Table 5 we may observe that a number of I items appear to be substantially related to Factor VI.

As a matter of fact, there is a whole cluster of Information items in Table 5, including all of the most difficult items for our sample as well as some of those that were very easy, which have about equal loadings on Factors III, V, and VI. It is notable and probably not coincidental that these are the same three factors for which the data dictated obliqueness, and which were rotated to yield *negative* correlations with one another. If the same oblique transformation were applied to Table 5, all of these loadings would simply be *in*creased. All these results can be accounted for by positing that the factors are normally nearly orthogonal, but that our sample has been severely selected in terms of a composite of the three factors. The existence of numerous items that measure this particular composite establishes that tests can be built to accomplish such selection. Furthermore we know that our sample is representative of only a limited range of scholastic aptitude, as indicated by scores obtained on the College Board's Scholastic Aptitude Test; it does not seem an unreasonable hypothesis to identify scholastic aptitude as a composite depending largely on Factors III, V, and VI, provided each of these factors is itself interpreted in the broad fashion outlined above. If this is true, then "scholastic aptitude" and "mental capacity" as we defined it in connection with Factor I are not equivalent concepts.

## SUMMARY AND CONCLUSIONS

Factor analysis is applied to a matrix of item intercorrelations based on the WAIS Information and Arithmetic subtests, using a sample of 228 college-

oriented males. Six factors are found and speculatively interpreted as follows:

I: *General Information*: This factor is best measured by information items that are non-specific in response requirement, and "historical" in their content. Scores on this factor may assess "mental capacity" in adults.

II: *Contemporary Affairs*: This factor is best measured by information items that have not necessarily been true for a long time, including the geographical items from the WAIS.

III: *Cultural Knowledge*: This factor is measured by information items of specific literary or religious interest, or which are normally acquired through reading.

IV: *Scientific Generalizations*: While this factor is well measured by appropriate information items, it is also involved in the performance of relatively complex mental arithmetic problems to which formulae can be applied, or which require a sequence of elementary operations.

V: *Numerical Information*: This factor may be directly measured by specific, role-appropriate information items, but is also involved in WAIS arithmetic items requiring the "conversion of units."

VI: *Numerical Operations*: This well-known factor depends on facility in the manipulation of familiar symbols according to simple rules, and may assess a form of "ideational discipline."

The Information subtest of the WAIS appears to depend primarily on Factors I, II, and III, with some items highly loaded on Factors IV and V. The Information subtest of the WB-I is similar except for the notable absence of Factor IV. The Arithmetic subtests of both the WB-I and the WAIS appear to depend primarily on Factor VI, with most items also loaded on Factor V, and most difficult items also loaded on Factor IV. Scholastic aptitude appears to be a composite of Factors III, V, and VI.

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